

AN IDEALISED BOUNDARY CONDITION FOR THE MESO-SCALE ANALYSIS OF TEXTILE IMPREGNATION PROCESSES

Wouter J.B. Grouve^{1,2}, Remko Akkerman¹

¹ *Faculty of Engineering Technology, Chair of Production Technology,
University of Twente, P.O. Box 217 7500AE Enschede, The Netherlands*

² *Corresponding author's Email: w.j.b.grouve@utwente.nl*

SUMMARY: The dual porosity of textile reinforcements, typically used in fibre reinforced plastics, poses a challenge in terms of permeability modelling. The influence of the dual porosity is found to be restricted to the interaction between the inter- and intra-bundle flow. It is proposed to account for this interaction by the application of a slip boundary condition on the bundle surface in a meso scale permeability model. The appropriate slip coefficient, for flow transverse to the bundle, is determined by solving the Stokes equations in a micro scale domain consisting of a single filament. A master curve is derived, relating the intra-bundle fibre volume fraction and slip coefficient. Application of the proposed slip coefficient on the bundle surface yields improved results compared to the use of a no-slip boundary condition. Future work focuses on the determination of a slip coefficient master curve for flow parallel to the bundles.

KEYWORDS: permeability, dual scale porosity, slip boundary condition, transverse flow

INTRODUCTION

The production of fibre reinforced plastic composites often involves the infiltration of a viscous resin into a reinforcing fibrous fabric. The flow physics in these production processes are typically simulated using Darcy's Law [4], given by

$$\mathbf{u} = -\frac{\mathbf{K}}{\mu} \nabla p \quad (1)$$

in which \mathbf{u} and p are the superficial velocity and pressure respectively, and μ is the fluid viscosity. The complicated interaction between fluid and fibrous reinforcement is lumped into the permeability tensor \mathbf{K} . Accurate permeability data are therefore essential in the simulation and optimisation of these flow processes. The permeability tensor is generally obtained by

experimental means. However, since such experiments are laborious, suffer from experimental scatter and have little predictive value, a modeling approach is desirable.

The main requirement in developing predictive permeability models is to consider the fabric architecture carefully. Recent developments, especially in CFD simulation software, have contributed considerably to the quality of the permeability predictions. Detailed architectural information, such as geometric shape, weave pattern and packing characteristics of the fabric can all be accounted for. The preform heterogeneity, however, still provides a challenge. The reinforcing fabric consists of bundles, which themselves comprise small filaments. Two length scales can therefore be identified: the meso-scale considers flow in the inter-bundle space, while the micro scale considers fluid flow in the intra-bundle space. The fabric permeability is therefore influenced by the following factors: (1) inter-bundle flow, (2) intra-bundle flow and (3) the interaction between inter- and intra-bundle flows.

The influence of the intra-bundle porosity on the global fabric permeability has been investigated by a number of authors. The majority of methodologies treat the inter-bundle space and bundle as separate domains, solving the Stokes equation in the inter-bundle domain and either Darcy's Law or Brinkman's equation [3] in the intra-bundle domain. Amico and Lekakou [1] applied Darcy's Law for the intra-bundle flow. A partial slip condition, as proposed by Beavers and Joseph [2], is required at the bundle interface to match fluid velocities and shear stresses. This problem can be circumvented by applying Brinkman's equation in the intra-bundle flow domain. Phelan Jr. and Wise [8] adopted this approach and derived a semi-analytical model to investigate the effect of bundle shape and intra-bundle permeability in a square packed bed on the overall permeability.

Nordlund [9] proposed an alternative method and modified the boundary conditions in an inter-bundle flow model to include intra-bundle flow. The permeability of multi-scale reinforcements can then be determined using a simplified fluid domain, consisting of the inter-bundle regions only. The influence of flow through the bundles is represented by slip velocities at the bundle surface parallel to the pressure gradient, while fluid flux boundary conditions are set on the interfaces perpendicular to it. The slip velocities are approximated by solving the one-dimensional Stokes equation for the inter-bundle flow and, again, Brinkman's equation for intra-bundle flow.

The requirement to prescribe a physically realistic value for the intra-bundle permeability can be seen as a disadvantage of the abovementioned approaches. Furthermore, the suitability of Brinkman's equation to model flow inside the bundles has been questioned by Larson and Higdon [5]. Papathanasiou et al. [7] solved the Stokes equation in both the inter- and intra-bundle domain, considering each filament individually. It was observed that even at intra-bundle porosities as high as 50 % the bundle behaves largely as an impenetrable entity, with only a small part of the fluid travelling through the bundle. The influence of the intra-bundle porosity can thus be restricted to the interaction between inter- and intra-bundle flow, while the actual intra-bundle flow is negligible.

The large variety in different fabric types calls for fast permeability solvers. This imposes severe restrictions on the modelling approach. A modification of the boundary conditions in an inter-bundle flow solver, as was performed by Nordlund [5], appears therefore promising. In the present work the two dimensional Stokes equations are solved in the micro-scale domain to yield

a slip boundary condition for the flow transverse to fiber bundles. This boundary condition can then be applied in the inter-bundle flow model, without having to solve the intra-bundle flow using Darcy's Law or Brinkman's equation.

SLIP BOUNDARY CONDITION

The interaction between inter and intra-bundle flows will be accounted for by applying a slip boundary condition on the bundle surface in an inter-bundle domain permeability model. The main requirement is that the resulting fluid velocity profile agrees with the profile for the full dual scale case. In this section a slip boundary condition similar to Beavers and Joseph [2] is introduced. A micro-scale unit cell, in which the Stokes equations are solved, is introduced to obtain the slip coefficients for viscous flow transverse to the fiber bundles.

Boundary Slip Condition

Consider the Poiseuille Flow between two plates as shown in Fig. 1. A slip condition, similar to Beavers and Joseph [2], is defined:

$$u(y) = \beta \cdot \frac{du}{dy} \quad (2)$$

The slip coefficient β in case of flow between the two plates at $y = \delta$ yields:

$$\beta(\delta) = \frac{u(\delta)}{du(\delta)/dy} = \frac{\frac{1}{2}\delta^2 - H\delta}{\delta - H} \quad (3)$$

The coefficient is only a function of location δ and the domain height H , and thus independent of viscosity or pressure gradient. The slip condition can be applied on a 'fictive' boundary in order to describe the fluid velocity field, as shown in Fig. 2.

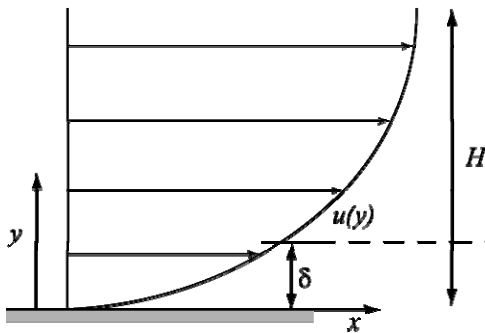


Fig. 1 Flow between two parallel plates.

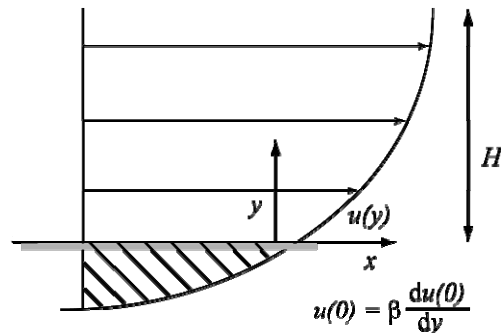


Fig. 2 Flow between two parallel plates applying a slip boundary condition.

The resin flow around bundles is influenced by the interaction effects between inter- and intra-bundle flow. Solving the Stokes equation in a domain consisting of individual filaments is impractical due to the cpu time required. In an alternative approach, analogous to the above mentioned example, the interaction effects can be incorporated in a meso scale model by applying a slip condition, i.e., Eqn. 2, on the bundle boundary. The appropriate slip coefficient β is determined in a micro-scale analysis.

Slip Coefficient for Flow around Bundles

The slip coefficient on the bundle surface is obtained by solving the Stokes Flow equations in a micro-scale unit cell, depicted in Fig. 3. The unit cell represents the surface of a bundle consisting of square packed filaments, with $2H$ the distance to a neighbouring bundle. The following boundary conditions have been applied on the unit cell: symmetry on the upper boundary, no slip on the lower boundary and a zero vertical velocity component and zero horizontal velocity gradient on the left and right boundary. A pressure difference is imposed on the left and right domain boundary. A finite difference based multigrid solver [6] is applied to solve the Stokes equations in the domain.

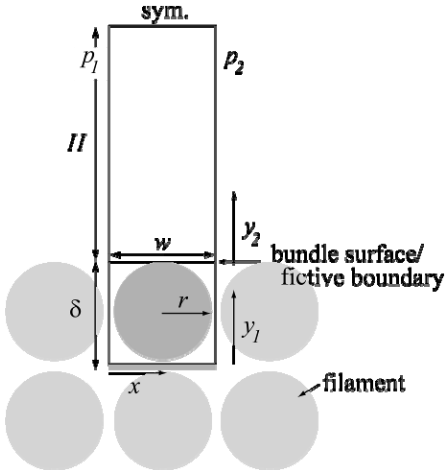


Fig. 3 Micro-scale unit cell.

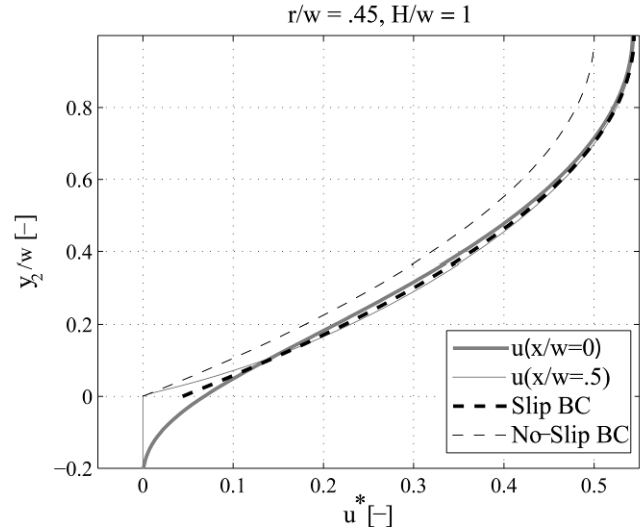


Fig. 4 Horizontal velocities in the unit cell.

The analysis has been non-dimensionalised by introducing $y^* = y/w$, $H^* = H/w$, $r^* = r/w$ and $x^* = x/w$. The dimensionless fluid velocity is then defined as:

$$u^*(x^*, y^*) = -\frac{\mu}{w^2 \frac{dp}{dx}} u(x, y) \quad (4)$$

The dimensionless bundle slip coefficient is defined as:

$$\beta^* = \frac{\bar{u}^*}{\bar{u}_y^*} = \frac{1}{w} \beta \quad (7)$$

in which the averaged dimensionless velocity on the bundle surface equals:

$$\bar{u}^* = \frac{1}{w} \int_{x^*=0}^{x^*=1} u^*(x^*, 0) dx^* \quad (5)$$

The averaged dimensionless velocity derivative on the bundle surface is given by:

$$\bar{u}_y^* = \frac{1}{w} \int_{x^*=0}^{x^*=1} \frac{du^*(x^*, 0)}{dy^*} dx^* \quad (6)$$

Fig. 4 shows the dimensionless horizontal fluid velocity u^* as function of the dimensionless position y/w . The velocity profiles at two different locations, i.e. $x/w = 0$ and $x/w = .5$, are plotted. The velocity field is reproduced in an inter-bundle domain analysis using both a no-slip boundary condition and the proposed slip boundary condition. The application of the slip boundary condition gives an excellent representation of the actual velocity field. The volume flux in the inter-bundle domain, applying the slip boundary condition, is given by:

$$Q_\beta^* = \int_0^{H/w} u^* dy^* = \frac{1}{3} H^{*3} + \beta^* H^{*2} \quad (8)$$

The difference between the volume flux based on the original numerical solution and the estimated results based on the slip coefficient, for the case $H/w = 1$, is less than 0.2%. The slip coefficients for distinct filament radii will be determined in the following section.

BUNDLE SLIP COEFFICIENT

Typical results for the particular case of $r/w = .45$ are determined. The slip coefficient is as function of the domain height H/w is plotted in Fig. 5. It can be seen that as H/w increases the slip coefficient β approaches a limit. The domain height $2H/w$ represents the gap between two adjacent bundles, with w the filament size. The obtained data is fitted with a asymptotic function:

$$\beta^* = \frac{c_1 + c_2 H^*}{H^*} \quad (9)$$

The limit value of the function is given by:

$$\beta_{\lim}^* = \lim_{H^* \rightarrow \infty} \beta^*(H^*) = c_2 \quad (10)$$

The space between the individual filaments governs the apparent slip velocity. As a result, the way in which the individual filaments are packed inside the bundle is not of importance. The velocity profiles for the flow over a bundle consisting of square and hexagonal, obtained by using an alternative unit cell, packed filaments are plotted in Fig. 6. The figure shows that, indeed, the velocity profiles at $x/w = 0$ for the two cases virtually coincide.

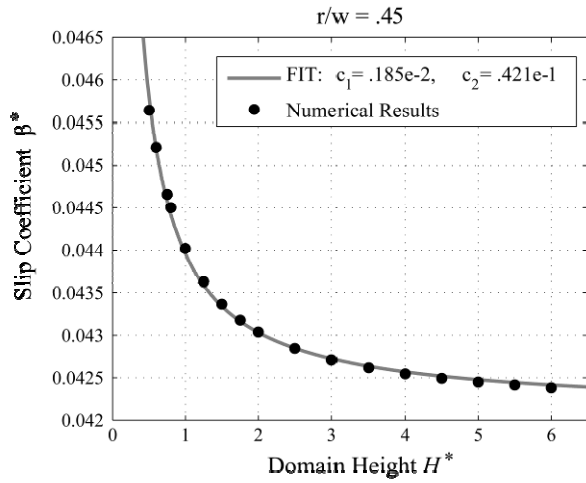


Fig. 5 Slip coefficient as function of domain height, for the case $r/w = .45$.

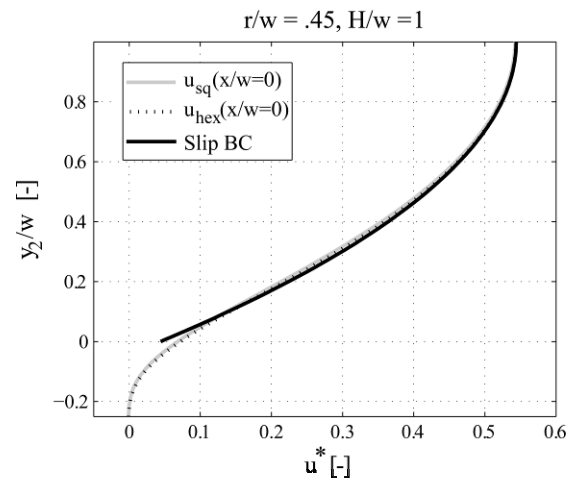


Fig. 6 Horizontal velocity profiles in a unit cell for square and hexagonal packed filaments.

The slip coefficient as function of fibre volume fraction, for distinct packing characteristics, can thus be determined with only one analysis. The limiting slip coefficients β_{lim}^* for different relative filament radii r/w and accompanying fibre volume fraction, for hexagonal and square packing, are plotted in Fig. 7. The data is fitted with a linear function to obtain a master curve:

$$\beta_{lim}^* = -.025 \cdot r^* + .031 \quad (11)$$

The master curve provides a slip coefficient which can be used in a meso scale permeability model in order to incorporate the interaction effects between inter- and intra bundle flow.

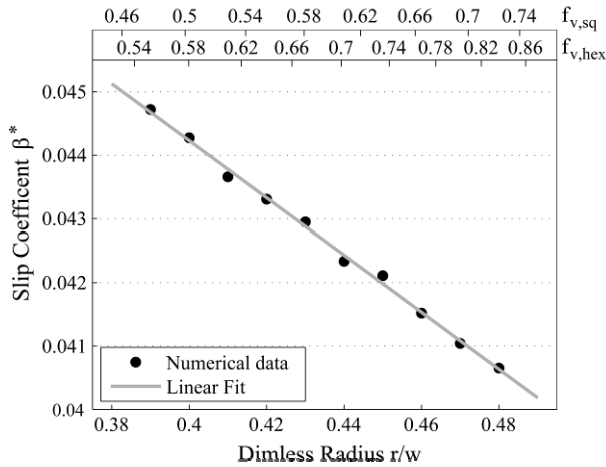


Fig. 7 Slip coefficient as function of domain height, for the case $r/w = .45$.

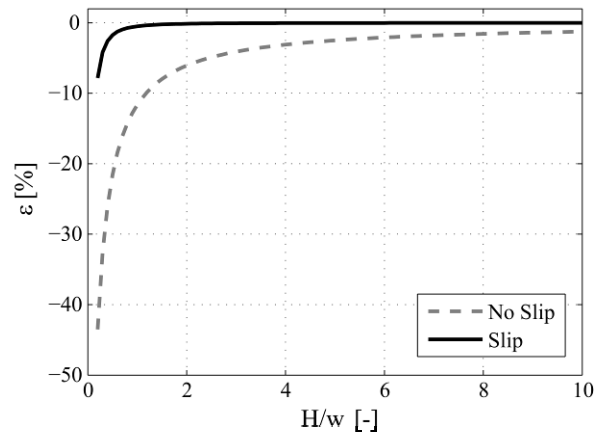


Fig. 8 Relative error in volume flux as function of dimless. domain height H^* .

The use of the limit values β_{lim}^* , shown in Fig. 7, introduces an error in the volume flux in case the distance between the bundles, i.e. H/w decreases. The relative error using the slip boundary condition has been defined as:

$$\varepsilon_{\beta,\text{lim}} = \frac{Q_{\beta,\text{lim}}^* - Q_{\beta}^*}{Q_{\beta}^*} \times 100\% \quad (12)$$

The volume flux Q_{β} is defined in Eqn. 8 and uses the slip coefficient curve, Fig. 5, while the volume flux $Q_{\beta,\text{lim}}$ is based on the limiting slip coefficient:

$$Q_{\beta,\text{lim}}^* = \int_0^{H/w} u^* dy^* = \frac{1}{3} H^{*3} + \beta_{\text{lim}}^* H^{*2} \quad (13)$$

For comparison, the relative error using a no-slip boundary condition is defined as:

$$\varepsilon_{\text{no slip}} = \frac{Q_{\text{no slip}}^* - Q_{\beta}^*}{Q_{\beta}^*} \times 100\% \quad (14)$$

Fig. 8 shows both errors as function of relative domain height H^* . The application of the proposed slip boundary condition is justified for values of $H^* > 1$, while for values $H^* > 8$ the no slip boundary condition on the bundles can be applied.

CONCLUSIONS AND FUTURE WORK

The Stokes equations were solved in a micro-scale domain, comprising one filament, to yield a meso scale slip coefficient for the case of viscous flow transverse to a fibre bundle. The interaction effects between the inter- and intra-bundle flow can be accounted for in a meso scale permeability model by applying a slip boundary condition using the obtained slip coefficients. A master curve was derived, relating the intra-bundle fibre volume fraction and slip coefficient. Application of the proposed slip coefficient yields improved results compared to a no-slip boundary condition.

Current research focuses on obtaining the bundle slip coefficient master curve for flow parallel to the fibre bundles. The Stokes equations, which in this particular case simplify to the Poisson problem, are again solved in a micro-scale domain. Future work will concentrate on implementing the master curves in a, finite difference based, multigrid permeability solver [6].

ACKNOWLEDGEMENTS

This work is performed with the support of from the Netherlands Agency for Aerospace Programmes (NIVR) and Ten Cate AC. This support is gratefully acknowledged by the authors.

REFERENCES

1. S. Amico and C. Lekakou, "Flow Through a Two-Scale Porosity, Oriented Fibre Porous Medium", *Transport in Porous Media*, Vol. 54, 2004, pages 35-53.
2. G.S. Beavers and D.D. Joseph, "Boundary Conditions at a Naturally Permeable Wall", *Journal of Fluid Mechanics*, Vol. 30, no. 1, 1967, pages 197-207.
3. H.C. Brinkman, "A Calculation of the Viscous Force Exerted by a Flowing Fluid on a Dense Swarm of Particles", *Applied Scientific Research*, Vol. 1, no. 1, 1949, pages 27-34.
4. H. Darcy, "Les Fontaines Publiques de la Ville de Dijon", *Delmont*, Paris, 1856.
5. R.E. Larson and J.J.L. Higdon, "Microscopic Flow Near the Surface of Two-Dimensional Porous Media. Part 1. Axial Flow", *Journal of Fluid Mechanics*, Vol 166, 1986, pages 449-472.
6. R. Loendersloot, "Permeability Prediction Using a Multigrid Method", *Proceedings of FPCM-9*, Montréal, Canada, 2008.
7. T.D. Papathanasiou et al., "On the Effective Permeability of Square Arrays of Permeable Fiber Tows", *Int. J. Multiphase Flow*, Vol. 23, no. 1, 1997, pages 81-92.
8. F.R. Phelan Jr. and G. Wise, "Analysis of Transverse Flow in Aligned Fibrous Media", *Composites Part A*, Vol. 27A, no. 5, 1995, pages 755-774.
9. M. Nordlund, "Permeability Modelling and Particle Deposition Mechanisms Related to Advanced Composite Manufacturing", PhD. Thesis, *Luleå University of Technology*, 2006.